\textbf{\textit{α}-particle Optical Potentials for Nuclear Astrophysics (NA) and Nuclear Technology (NT)}

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\textbf{Abstract.} The high precision of recent measurements for low-energy $\alpha$-particle elastic-scattering as well as induced-reaction data makes possible the understanding of actual limits and possible improvement of the global optical model potentials parameters. Involvement of recent optical potentials for reliable description of both the elastic scattering and emission of $\alpha$-particles, of equal interest for nuclear astrophysics (NA) and nuclear technology (NT) for fusion devices, is discussed in the present work.

\textbf{Keywords:} Optical model, Phenomenological Potential, Alpha-particle Elastic Scattering.

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\section*{INTRODUCTION}

In the last decade, several approaches have been developed in order to solve two questions which are still open concerning the optical model potential (OMP) analyses for $\alpha$-particles at low energies (e.g. [1] and Refs. therein), namely (i) the OMP parameter sets obtained from $\alpha$-particle elastic scattering at high energies (>80 MeV) do not describe either the low-energy (<40 MeV) elastic scattering or the complete fusion data, and (ii) the statistical $\alpha$-particle emission is underestimated by the OMPs that account for elastic scattering on the (cold) ground-state nuclei. In the latter case, the need for new physics in potentials to describe nuclear de-excitation within the statistical model calculations was pointed out [2]. It was thus suggested that particle evaporation occurs from a transient nuclear stratosphere of the emitter nucleus, with a density that differs from cold nuclei [3] and which has not yet relaxed to the density profile expected for complete equilibration [4]. Therefore, effects due to changes of the nuclear density at a finite temperature have been considered within the double folding formalism [5] of the alpha-nucleus real part of the optical potential.

At the same time, the high precision of recent measurements of low-energy $\alpha$-particle elastic-scattering as well as induced-reaction data [6] makes possible, for the global OMP parameters formerly obtained [1,7,8], the understanding of actual limits and possible improvement [9], of equal interest for nuclear astrophysics (NA) and nuclear technology (NT) for fusion devices. A better knowledge of this issue would be also very convenient [10] for additional insight on basic nuclear properties as the moment of inertia which determines the spin distribution of the nuclear level density.
FORMER ELASTIC SCATTERING ANALYSIS

As a matter of fact, the widely-used phenomenological OMP parameters are mainly derived from the analysis of elastic-scattering angular distributions, which are however ruled out below the Coulomb barrier $B$. Moreover, results from the analysis of the low-energy elastic-scattering data suffer from discrete and continuous ambiguities in the OMP parameters, whose uncertainties vary for various target nuclei and for different incident energies due to the precision of the data analyzed. Therefore, in order to avoid too much phenomenology in the description of these data, numerous attempts have been made to replace the phenomenological real potential of Woods-Saxon (WS) type by a more microscopic $\alpha$-nucleus potential using an effective interaction. The double-folding (DF) model [11] has become widely used with an effective nucleon-nucleon (NN) interaction folded with the mass distributions of both the target nucleus and the projectile. While the M3Y Reid [12] and Paris [13] are the most familiar interactions [11,14,15], a density dependence has been also incorporated especially for the description of refractive nuclear scattering at higher energies. In this case the scattering data are sensitive to a wide radial domain and various specific forms have been adopted in the M3Y-interactions ([16] and Refs. therein) for the corresponding density-dependent factor so as to reproduce consistently the same saturation properties of nuclear matter in Hartree-Fock calculations. However, the M3Y interaction can be used only to obtain the real potential, and the imaginary term must be parameterized independently (e.g., [15]) or simply taken from a phenomenological OMP [11]. As a first result, the former approach may reduce the number of the OMP parameters and corresponding uncertainties, with the success proved in the description of the elastic scattering of many systems [15].

As stated from the beginning in Ref. [1], we started with the analysis of the $\alpha$-particle elastic scattering alone, for nuclei in the mass region $A\sim100$ and energies from $\sim14$ to 32 MeV, while an eventual failure to describe reaction data remained to be understood later. Thus, we did not take into account either the available experimental $\alpha$-induced or the $(n,\alpha)$ reaction cross sections, to avoid additional difficulties because of the remaining parameters needed in statistical model calculations [17].

First, a semi-microscopic analysis based on the DF model for the $\alpha$-particle elastic scattering on $A\sim100$ nuclei at energies below 32 MeV has been carried out. The energy-dependent phenomenological imaginary part for this semi-microscopic OMP was obtained, making use also of the dispersive correction to the microscopic real potential. This imaginary potential has then been introduced within a complete phenomenological analysis of the same data basis. A regional optical potential (ROP) parameter set for low-energy $\alpha$-particles has thus been obtained for nuclei in the mass range $A\sim100$. Moreover, we found that the differences between the semi-microscopic and phenomenological real potentials come mainly from their different radial dependence, as can be seen in Fig. 10 of Ref. [1]. However these potentials are almost identical in the tail region, i.e. for the depth absolute values $\sim10$ MeV [18], and extend over a wider radial range for lower incident energies which may explain the actual similar description of the elastic-scattering data at low energies.

The same ($\alpha,\alpha$) semi-microscopic and phenomenological analysis was then extended to medium-mass $A\sim50-120$ [7] and heavy $A\sim132-209$ [19] nuclei.
ROLE OF HIGH-ACCURACY (α,α) DATA FOR OMP UPGRADE

The experimental data that were available and used to obtain the optical potential of Ref. [1] have had accuracies of the order of 10-15%. Rather soon after that, Galaviz et al. [20] reported new experimental information of the α-particle elastic scattering on $^{112,124}$Sn at energies just below and above the Coulomb barrier, with typical uncertainties below 3-4%. Moreover, they have shown that data obtained along an isotopic chain with so high precision, that is actually enhanced by a factor of ~5 with respect to the former data basis, may also supply further information about the α-nucleus potentials. Thus, the ratio of the elastic-scattering cross sections of $^{112}$Sn and $^{124}$Sn at the α-particle energy of 19.5 MeV, that rises to ~1.4 at backward angles, shows an oscillation pattern at these angles. Moreover, Galaviz et al. [20] found that global α-nucleus potentials including ROP of Ref. [1] fail to reproduce this pattern. Since a global OMP must be able to describe the scattering cross-section data along an isotopic chain in order to demonstrate its reliability when extrapolating to unstable nuclei, they concluded that the use of these global potentials in the extrapolation to more proton-rich species should be questioned. On the other hand, we have shown [21] that it is possible to improve the ROP parameter set [1] by looking for the origin of the respective deviations, and using both the corresponding former local parameter basis and the parameter sets obtained through a similar analysis of the new data.

Thus, we have shown [21] that the oscillation pattern of the ratio of the two experimental cross sections, divided by the Rutherford cross section, as well as the deficiency of ROP predictions was hidden by the uncertainties of the earlier data involved within former analysis [1]. It is true that the systematic errors included in the older error bars resulted at least in part from overall-normalization uncertainties, so that our enlarged estimates may not really be representative of the actual data errors used effectively within the $\chi^2$ analysis. However it was obvious that the new features of the data of Galaviz et al. as well as the inaccuracy of the previous global OMPs are matching just their magnitude. Moreover, the additional analysis of the new data by means of the above-mentioned method, used for the ROP setting up [1], led to the WS potential parameters that made possible a properly improved agreement of the calculated and the new measured data while the oscillations of the ratio of angular distributions at 19.5 MeV are almost well reproduced (Fig. 1 of [21]) in a similar way to that of the original local potentials [20]. It should be noted in particular that, while the latter surface imaginary potential depths have values of a couple of hundred MeV, the corresponding values of our parameters are of several MeV.

Furthermore, the new local parameter sets were used with respect to the corresponding predictions of ROP [1] in order to learn more about the shortcomings of the latter. Thus, a more accurate form of the real potential depth has resulted, already leading to the correct maxima and minima position for the cross-section ratio. Additional, actually largest, parameter change of surface imaginary potential depth led to a revised version of the ROP parameter set that provides also calculated cross-section ratio oscillation [21] closer to the measured data trend [20].

Nevertheless, the largest parameter change found necessary for the OMP imaginary part should be also related to the need of an accurate energy-dependent WS imaginary part for the dispersive corrections to the microscopic DF real potential, within a semi-
microscopic OMP analysis (e.g., [5] and Refs. therein). The rather scarce data basis used for the former ROP [1] establishment has been obviously far by meeting this demand. This point was one of the grounds for the extension [7] of this semi-microscopic OMP analysis on lighter \( A \sim 60 \) nuclei, in order to use the corresponding larger data basis of the \( \alpha \)-particle elastic scattering at the same time with a reanalysis of all available \((\alpha,\alpha)\) data within the range \( A \sim 50-120 \).

On the other hand, extrapolation to very low energies of global potentials from higher energies is not appropriate due to the strong change in the number of open reaction channels close to \( B \). Thus it results a strong energy dependence of the OMP imaginary part [7], which takes into account all non-elastic interactions globally and accounts for the effect of removing flux from the elastic channel. This point was proved in the meantime also for ROP through its use for analysis of various \((\alpha,x)\) reaction cross sections around and below \( B \), i.e. outside the energy range involved for ROP setting up. Therefore, the \( \alpha \)-particle optical potential below \( B \) can be validated only by analyzing reaction cross sections too. Following this third step of OMP analysis, an extended ROP parameter set has been finally obtained within the mass range \( A \sim 50-120 \) [7], that was nearly at the same time referred with the 3rd phase of the Reference Input Parameter Library (RIPL-3) data base [22]. However, before going into a discussion of the \((\alpha,x)\) reaction cross-section analysis and results consequently obtained, we would like to note several more features related to the \( \alpha \)-particle elastic-scattering analysis.

**ACTUAL STATUS OF \((\alpha,\alpha)\) DESCRIPTION BY GLOBAL OMPs**

The extended ROP [7] has been already involved with analysis of subsequent measurements of \( \alpha \)-particle elastic-scattering angular distributions [23], including the ratio of the measured cross sections of the \(^{89}\text{Y}(\alpha,\alpha)^{89}\text{Y} / ^{92}\text{Mo}(\alpha,\alpha)^{92}\text{Mo} \) and \(^{106,110,116}\text{Cd}(\alpha,\alpha)^{106,110,116}\text{Cd}\) reactions that show an oscillation pattern at backward angles. The corresponding predictions have been better than ones provided by other global OMPs, with inherent limits in front of the local potentials established through the related analysis except for the unphysical depth of the latter imaginary potential depths [23]. Actually, one may find similar lack of agreement between the regional predictions [7] and local fits of Refs. [23] as that between the regional and local parameter sets in Ref. [7].

A quite distinct case is that of a recent measurement and analysis of \( \alpha \)-particle elastic scattering on \(^{106}\text{Cd},^{118}\text{Sn}, \) and \(^{120,124,126,128,130}\text{Te}\) target nuclei, at energies between 17 and 27 MeV [24]. The local OMP parameters sets obtained through the fit of these data have been compared with four widely used global or regional potentials, in the respective authors’ opinion, one of them being not the extended ROP of Refs. [7,22] but the former ROP of Ref. [1], even without the changes [21] given just next to the former cross-section ratio analysis [20]. In these conditions the conclusion that the OMP of Ref. [1] dramatically over-predicts the diffraction oscillations has been achieved [24]. However, the agreement of the cross sections measured and predicted but using the same extended ROP [7,22] as within Ref. [23], shown here in Fig. 1, may lead to the same comments as within the first paragraph of this section. The same
FIGURE 1. (Color online) Comparison of experimental angular distributions of the elastic scattering of $\alpha$-particles at energies between 17 and 27 MeV on $^{106}$Cd, $^{118}$Sn, and $^{120,124,126,128,130}$Te target nuclei [24], divided by the Rutherford cross section $\sigma_R$, to predictions of the optical potential of Refs. [7,22].
conclusion is valid for ratio of the $^{130}\text{Te}(\alpha,\alpha)^{130}\text{Te}/^{120}\text{Te}(\alpha,\alpha)^{120}\text{Te}$ reaction cross sections (Fig. 13 of Ref. [24] and bottom-right corner of Fig. 1).

The correct comparison of the measured and not former but latter predicted data is most useful for improvement of global OMP parameter sets, taking into account, e.g., the similar description of the $(\alpha,\alpha)$ data on the lightest isotopes and at lowest energies, and the agreement decrease with the mass and energy increase. At the same time it is not possible to get a definite conclusion on the parallel comparison, with the new measured data, of the more recent alternate three global semi-microscopic OMPs (I-III) [17] predictions since only the results obtained with their purely volume imaginary term potential I are shown in Figs. 12 and 13 of Ref. [24].

$(\alpha,\alpha)$ AND $(\alpha,X)$ COMPLEMENTARY ANALYSES

On the other hand, because of their scarcity and small size due to the Coulomb barrier penetration, the analysis of the $(\alpha,x)$ reaction cross sections is challenging. However, besides its basic interest, the accurate account of the $\alpha$-particle OMP required by this analysis is highly required by many nuclear astrophysics applications as well as by the nuclear engineering design of fusion test facilities.

Therefore, following the semi-microscopic and the phenomenological analyses of $(\alpha,\alpha)$ angular distributions, carried out usually above the Coulomb barrier, an ultimate assessment of $(\alpha,\gamma)$, $(\alpha,n)$ and $(\alpha,p)$ reaction cross sections concerned target nuclei from $^{45}\text{Sc}$ to $^{118}\text{Sn}$ and incident energies below $\sim \text{12 MeV}$ [25], i.e. below $B$. The former diffuseness of the real part of the WS optical potential as well as the surface imaginary potential depth, obtained through the elastic-scattering analysis, have been found responsible for the actual difficulties in the description of the reaction data [7]. Furthermore, these OMP parameters were modified in order to obtain an optical potential which describe equally well both the low energy elastic scattering and the $\alpha$-particle induced reaction data. For the rest of statistical model (SM) parameters we used consistent sets established by analyzing various independent experimental data for all stable isotopes of $\text{V, Mn, Co, Ni, Cu}$ [26], $\text{Mo}$ [27], and $\text{Pd, Sn and Te}$ [28], which finally allowed us to focus on the uncertainties of the $\alpha$-particle OMP. While a suitable description of the $(\alpha,n)$ reaction cross sections was found for lighter target nuclei with $A \leq 54$, a major overestimation of $(\alpha,\gamma)$ reaction cross sections resulted for $^{62,64}\text{Ni}$, $^{63}\text{Cu}$, $^{96,98}\text{Ru}$, $^{106}\text{Cd}$ and $^{112,118}\text{Sn}$. This behavior was related to energies around the Coulomb barrier, for the former mass range, and clearly below it for the latter. A suitable description of the $(\alpha,x)$ reaction data below $B$ was no longer possible by means of the optical potential provided by the elastic-scattering data analysis above this barrier, a modified surface imaginary potential being necessary. A reliable endorsement of the final ROP below the Coulomb barrier has finally been given by its additional, straightforward and successful use in calculations of reaction cross sections for the target nuclei $^{44}\text{Ti}$, $^{55}\text{Mn}$, $^{56}\text{Fe}$, $^{59}\text{Co}$, $^{58}\text{Ni}$, $^{65}\text{Cu}$, and $^{70}\text{Ge}$ [7].

Moreover, an additional analysis of $(\alpha,\gamma)$ and $(\alpha,n)$ reaction cross sections close to $B$, measured in the meantime for $^{92,94}\text{Mo}$, $^{112}\text{Sn}$, and $^{113}\text{In}$ nuclei [6], reconfirmed this OMP appropriateness [9].
In fact, the drop of the surface imaginary potential depth with the decrease of α-particle energy below the Coulomb barrier, necessary for the (α,x) reaction data account, is in line with the strong change of the number of open reaction channels within this energy range. It can also explain the even worse results of the former α-particle ROP based on the elastic-scattering analysis alone [1], when used in the (α,x) reaction data analysis, as opposed to the well-known four parameter global potential of McFadden and Satchler [29] and related OMPs [30] which have only a volume imaginary potential with a constant depth. The lack of an energy dependent surface component thus prevents larger negative effects by extrapolation below the Coulomb barrier. On the other hand, becoming aware of the changes of α-particle OMPs obtained through elastic-scattering analysis alone, in order to describe the (α,x) reaction data as well, the difference between the α-particle OMPs in the entrance and exit channels could be easier understood.

**EXTENSION TO THE MASS REGION 50<\(A\)<209**

While the energy dependence of the diffuseness \(a_R\) of the real part of optical potential and the depth \(W_D\) of derivative-shape surface imaginary potential had to be modified for \(E_{c.m.}/B<0.9\) in order to obtain an optical potential that describes equally well also the low energy α-particle induced reactions [7], it become necessary to determine if similar changes occur when the Coulomb barrier increases, for heavier target nuclei. First, an extension of the semi-microscopic analysis that uses the DF real potential for the whole mass region 50<\(A\)<209 nuclei and energies from ~8 to 50 MeV has also been carried out [19] with no DF adjustable parameter or normalization. The analysis included the available α-particle elastic-scattering angular distributions for 13 target nuclei from \(^{132}\)Ba until \(^{209}\)Bi. The results of the same two-step OMP approach [1,7], shown in Figs. 1 and 2 of Ref. [19], actually endorse the previous regional parameter set. Minor changes were necessary only for the depth and diffuseness of the optical-potential real part, with effects found to be larger only for \(A\geq132\).

Second, the above-mentioned WS parameters were used to carry out a first SM analysis of all available α-particle induced reaction cross sections on 15 nuclei with 121≤\(A\)≤197 [25]. One should note that the increased \(B\)-values associated with medium nuclei move the energy range of interest close to 20 MeV (Fig. 1 of Ref. [8]). Moreover, we have to limit our analysis to ~25 MeV because of the pre-equilibrium emission effects that may trigger additional problems for the SM calculated reaction cross sections. The (\(\alpha,\gamma\)) reaction data for \(A\geq121\), below the Coulomb barrier, were actually available only for \(^{127}\)I, \(^{136}\)Xe, \(^{139}\)La, \(^{144}\)Sm, and \(^{197}\)Au. Nevertheless, the overall good agreement between the measured and calculated cross sections [8] has confirmed the α-particle OMP obtained by elastic-scattering data analysis within the whole range 45≤\(A\)≤197 except for the lowest energy limit value of the surface imaginary potential depth \(W_D\). It has been rather well established by means of the (\(\alpha,\gamma\)) reaction study at α-particle energies where this reaction cross section stands actually for the α-particle total reaction cross section. Unfortunately, while there have been several \(A<120\) target nuclei (\(^{56}\)Fe, \(^{58,62,64}\)Ni, \(^{70}\)Ge, \(^{98}\)Ru, \(^{106}\)Cd, \(^{112}\)Sn) with measured (\(\alpha,\gamma\)) reaction data well described by the \(W_D\) value of ~3.5 MeV [7], only the available
data corresponding to the target nucleus $^{144}$Sm are useful in this respect for $A>130$. These data could be described by using a value $W_D \sim 1.5$ MeV [8]. As a result, the value $(2.5 \pm 1.0)$ MeV was adopted for the lowest-energy limit of this parameter. Further experimental reaction cross sections should provide better parameterization constraint and make possible a physical insight of the corresponding trend.

Otherwise, while there are more data below the Coulomb barrier for the $(\alpha,n)$ reaction than for the $(\alpha,\gamma)$ reaction, a suitable description of these reaction cross sections has been obtained within the critical energy range just above the threshold. To sum up, a larger overestimation of measured data may be generated through an extension to the lower energies of an optical potential that is established on the basis of only elastic-scattering data analysis and includes a surface imaginary part [8]. This component increases first with the energy increase, as more and more reaction channels are opened, but then decreases and eventually vanishes as the larger-energy $\alpha$-nucleus interactions take place to a greater extent inside the nucleus. Since the elastic-scattering data analysis just above the Coulomb barrier facilitates the description of the latter side of the surface imaginary-potential energy dependence, the extrapolation to much lower energies of this partial trend becomes unphysical and leads to a larger breakdown than its absence [8].

**EMITTED $\alpha$-PARTICLE OPTICAL POTENTIALS**

Since the phenomenological ROP [1], obtained by starting with a semi-microscopic analysis of the $(\alpha,\alpha)$ data around $A=100$, failed to describe the $(n,\alpha)$ reaction cross sections for the target nuclei $^{92,95,98,100}$Mo, a temperature dependence of the nuclear density within the DF formalism was considered [5]. Thus, it was shown that a temperature-dependent nuclear density distribution can be an important aspect to be included in statistical-model calculations even for a nuclear temperature $<2$ MeV. Actually, a similar description of the $(n,\alpha)$ reaction cross sections is provided by different OMPs which match each other in the outer limit of the nuclear surface as well as in its tail. It is astonishing that this result but smaller potential values characterize also the optical potentials which have been obtained by analyzing the $\alpha$-particle elastic scattering at low energies, i.e. the microscopic DF potential, the ROP [1], and the global OMP of McFadden and Satchler [29]. Therefore it seems that a basic difference exists, which is not explained by the microscopic potential. A possible explanation was found to be a temperature-dependent shape of the nuclear density, which produces decreased central values and a larger diffuseness of the DF real potential. By taking into account a correspondingly changed nuclear density distribution within calculation of the microscopic DF potentials, the new characteristics are able to fully describe the $(n,\alpha)$ reaction cross sections. Indeed the temperature dependence of the nuclear density distribution function could be the missing degree of freedom that needs to be included in conventional statistical model calculations even for nuclear temperature smaller than 2 MeV. Nevertheless, additional theoretical studies of the temperature effects on nuclear matter density would be quite helpful for the assumption of a realistic temperature dependence. However, the available data basis is still poor and further measurements with
increased incident energy accuracy will be particularly helpful for a further understanding of the interactions of low energy $\alpha$-particles.

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